

OVERVIEW OF NNLO QCD CORRECTIONS

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We discuss the motivation for making predictions for jet cross sections at next-to-next-to-leading order. We describe the theoretical ingredients needed for such a calculation and briefly review the progress in the field.

In the last decade there has been enormous progress in using perturbative QCD to predict and describe events containing jets. At the simplest lowest-order (LO) level, each jet is the footprint of a hard, well separated parton produced in the event. Although the predicted rate is sensitive to the choices of renormalization and factorization scales, qualitative comparisons of data and theory are generally very good. A more quantitative description is achieved by improving the theoretical prediction to next-to-leading order (NLO). This has the effect of reducing the dependence on the unphysical renormalization and factorization scales so that the normalization is more certain. Furthermore, the sensitivity to the details of the jet finding algorithm and the size of the jet is increased since now two partons may be combined to form the jet. However, for the most basic jet production processes such as $p\bar{p} \rightarrow \text{jet} + X$, $p\bar{p} \rightarrow V + \text{jet} + X$, $e^+e^- \rightarrow 3 \text{ jets}$ or $ep \rightarrow e + (2 + 1) \text{ jets}$, the experimental accuracy is such that even more precise theoretical predictions are required. In this talk, we review the recent progress made towards predicting jet cross sections at next-to-next-to-leading order (NNLO).

The addition of NNLO effects gives significant improvements over an NLO estimate. First, the dependence on the unphysical scales is significantly reduced. At NLO we find a reliable estimate of the cross section, while NNLO calculations yield a reliable estimate of the uncertainty in the cross section. For example, if we consider the single jet inclusive cross section for jets with $E_T = 100 \text{ GeV}$ and $0.1 < |\eta| < 0.7$ at $\sqrt{s} = 1800 \text{ GeV}$, the renormalisation scale dependence due to variations of a factor of two about $\mu_R = E_T$ is reduced from 20% to 9% to 1% as we move from

LO to NLO to NNLO. The NNLO estimate is obtained by including the renormalisation group predictable parts of the α_s^4 contribution and assuming that the (presently unknown) genuine NNLO contribution is zero. Other ingredients such as the factorisation scale and choice of parton density functions are kept fixed. Second, the sensitivity to the jet algorithm is further enhanced with up to three partons combining to form the jet. Radiation outside the jet is better described and more of the parton shower is explicitly reconstructed.

There are several ingredients necessary for NNLO calculations of, for example, jet production in hadron colliders. From the matrix element point of view we need

- the interference of the tree and two-loop amplitudes for the two parton final state,
- the square of the one-loop amplitude for the two parton final state,
- the interference of the tree and one-loop amplitudes for the three parton final state,
- the square of the tree amplitude for the four parton final state.

Each of these contributions is infrared divergent and a systematic procedure for analytically cancelling the infrared divergences between the tree-level $2 \rightarrow 4$, the one-loop $2 \rightarrow 3$ and the $2 \rightarrow 2$ processes needs to be established. In addition, for physical cross sections, the parton density functions are needed at NNLO which in turn requires knowledge of the three-loop splitting functions as well as a global fit to other observables (such as deep inelastic scattering and Drell-Yan) computed to NNLO. Finally, and most importantly for phenomenological applications, a numerical implementation of the various contributions must be developed.

Matrix elements

Techniques for computing multiparticle tree amplitudes for $2 \rightarrow 4$ processes, and the associated crossed processes, are well understood. For example, the helicity amplitudes for the six gluon $gg \rightarrow gggg$, four gluon-two quark $\bar{q}q \rightarrow gggg$, two gluon-four quark $\bar{q}q \rightarrow \bar{q}'q'gg$ and six quark $\bar{q}q \rightarrow \bar{q}'q'\bar{q}''q''$ have been computed in Refs.¹. Similarly, amplitudes for the one-loop $2 \rightarrow 3$ parton sub-processes $gg \rightarrow ggg$, $\bar{q}q \rightarrow ggg$, $\bar{q}q \rightarrow \bar{q}'q'g$, and processes related to these by crossing symmetry, are also known and are available in² respectively. However, the evaluation of the two-loop $2 \rightarrow 2$ contributions for QCD processes has been a challenge for the past few years, mainly due to a lack of knowledge about planar and crossed double box integrals that arise at this level.

In the massless parton limit and in dimensional regularisation, analytic expressions for these basic scalar integrals have now been provided by Smirnov³ and Tausk⁴ as series in $\epsilon = (4-D)/2$, where D is the space-time dimension, together with algorithms for reducing tensor integral to a basis set of known scalar (master) integrals⁵. This makes the calculation of the two-loop amplitudes for $2 \rightarrow 2$ QCD scattering processes possible.

The general strategy is to rewrite the interference of two-loop with tree graphs in terms of a limited number of master loop integrals, which can then be expanded as a series in $\epsilon = (4-D)/2$, so that

$$\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle = \sum_{i=1}^4 \frac{X_i}{\epsilon^i} + X_0. \quad (1)$$

After renormalisation, the infrared singularities indicated by poles in the dimensional regulator ϵ correspond to soft and/or collinear virtual emissions. The structure of these singularities can be predicted using the general factorisation formula of Catani⁶ and serves as a powerful check on the calculation. Following on from the pioneering work of Bern, Dixon and Ghinculov⁷ who calculated the two-loop contribution to the QED processes $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow e^-e^+$, the $\mathcal{O}(\alpha_s^4)$ contributions arising from the interference of two-loop and tree-level graphs for the QCD

processes of quark-quark, quark-gluon and gluon-gluon scattering⁸ have now been computed. Similar techniques can be applied to other two-loop processes with massless internal particles and on-shell external legs such as $gg \rightarrow \gamma\gamma$ or $q\bar{q} \rightarrow \gamma\gamma$.

Differential equation techniques have been developed for computing two-loop master integrals with off-shell legs⁹ and all of the planar and non-planar¹⁰ master integrals are now known with one off-shell leg. Such integrals are vital for basic scattering processes such as $Z \rightarrow q\bar{q}g$ and $ep \rightarrow (2+1)$ jets.

Infrared cancellation

The infrared singularities present in the virtual contributions (Eq. 1) must cancel against the contributions from the one-loop $2 \rightarrow 3$ processes when one particle is unresolved and the contribution from the tree-level $2 \rightarrow 4$ processes when two particles are unresolved. Unresolved particles are either soft or collinear with one of the other partons in the event and both of these configurations have the appearance of a $2 \rightarrow 2$ scattering. A systematic procedure for analytically carrying through the cancellation (as well as providing a series of counter terms that remove the divergence from the radiative contributions) has not yet been established. However, single and double unresolved limits of the matrix elements are well known and this may be a tractable problem. For example, in the limit where three particles are simultaneously collinear, the (colour ordered) tree amplitude undergoes a factorisation of the form¹¹

$$\mathcal{M}^{(0)}(\dots, a, b, c, \dots) \rightarrow P_{abc \rightarrow d} \mathcal{M}^{(0)}(\dots, d, \dots)$$

In other words, we find the tree level amplitude with particles a , b and c replaced with a single particle d multiplied by a single (universal) factor describing the splitting $d \rightarrow a+b+c$. All of the divergences associated with the collinear singularities are contained inside the splitting function $P_{abc \rightarrow d}$ and, when integrated over the triple collinear phase space yield singular contributions up to $1/\epsilon^3$. Similar factorisation formulae apply in the double soft¹² and soft-collinear¹¹ limits.

The one-loop amplitudes are also singular in the single unresolved limit - either due to two particles becoming collinear or a soft gluon. Once again there is a factorisation^{13,14} that yields a tree unresolved factor multiplied by a one-loop amplitude with one fewer external particle plus a loop unresolved factor multiplied by a tree amplitude.

Many of the analytic phase space integrations for the double unresolved and single unresolved loop contributions have already been studied in the context of $e^+e^- \rightarrow \text{photon} + \text{jet}$ at $\mathcal{O}(\alpha\alpha_s)$ ¹⁵ and Higgs production in hadron colliders¹⁶.

Parton density Functions

A further complication is due to initial state radiation. Factorization of the collinear singularities from the incoming partons requires the evolution of the parton density functions to be known to an accuracy matching that of the hard scattering matrix element. This entails knowledge of the three-loop splitting functions. At three-loop order, the even moments of the splitting functions are known for the flavour singlet and non-singlet structure functions F_2 and F_L up to $N = 12$ while the odd moments up to $N = 13$ are known for F_3 ¹⁷. The numerically small N_F^2 non-singlet contribution is also known¹⁸. Van Neerven and Vogt have provided accurate parameterisations of the splitting functions in x -space¹⁹ which are now starting to be implemented in the global analyses²⁰.

Outlook

While much work remains to be done, the current rate of progress suggests that numerical estimates of jet cross sections at NNLO may become available in the next couple of years. The

theoretical uncertainty at NNLO should be significantly reduced compared to NLO estimates enabling more stringent tests of QCD at short distances.

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